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Effects of dynamical Wilson fermions and the phase structure of compact QED_4†

A. Hoferichter[a], V.K. Mitrjushkin[b], M. Müller-Preussker[c] and H. Stüben[d]

[a] *DESY-IfH and HLRZ, Zeuthen, Germany*
[b] *Joint Institute for Nuclear Research, Dubna, Russia*
[c] *Institut für Physik,Humboldt-Universität zu Berlin, Germany*
[d] *Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany*

Abstract

By comparison of the quenched and full formulations of compact QED with Wilson fermions we single out the effects of dynamical fermions on the 'chiral transition line' within the confinement phase. It is shown that this line cannot correspond to the chiral limit of the theory for all values of the gauge coupling. This seems to imply the existence of tri-critical points in this theory, the phase structure of which has a close similarity to QCD at finite temperature.

1 Introduction

Wilson lattice fermions break chiral symmetry explicitly. Hopefully, it can be recovered by fine-tuning the bare parameters in the continuum limit. In the phase diagram the 'critical' line $\kappa_c(\beta)$ is associated with the chiral limit

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of the theory. For non-vanishing lattice spacing on this line only a partial restoration of chiral symmetry occurs.

In this letter we are concerned with the behavior of fermionic observables, in particular with the pseudo-scalar 'pion' mass close to $\kappa_c(\beta)$ in the confinement phase of compact QED. We confront the full theory with its valence approximation.

2 The Model, Observables and the Phase Structure

We consider the Wilson action for QED

$$S_W = S_G(U) + S_F(U, \bar{\psi}, \psi) \tag{1}$$

consisting of the standard plaquette compact $U(1)$ gauge action $S_G(U)$ and the fermionic part

$$S_F(U, \bar{\psi}, \psi) = \sum_{f=1,2} \sum_{x,y} \bar{\psi}_x^f \mathcal{M}_{xy}(U) \psi_y^f \ , \tag{2}$$

with the Wilson matrix $\mathcal{M}(U) = \hat{1} - \kappa \mathcal{D}(U)$,

$$\mathcal{D}_{xy} \equiv \sum_{\mu} \Big[\delta_{y,x+\hat{\mu}} P_\mu^- U_{x\mu} + \delta_{y,x-\hat{\mu}} P_\mu^+ U_{x-\hat{\mu},\mu}^\dagger \Big] \tag{3}$$

and $P_\mu^\pm = \hat{1} \pm \gamma_\mu$. We adopt the relation of the hopping parameter κ to the fermion mass $m_q = (1/\kappa - 1/\kappa_c(\beta))/2$.

The phase structure of the path integral quantized theory has been investigated in [1]. For full QED a diagram with four phases in the range $0 \leq \kappa < 0.3$ emerged (see Fig. 1). Whereas the Coulomb and confinement phases are quite well understood, the situation in the 'upper' areas deserves further study (see e.g. [2]). We mention that this phase diagram has similarities with that of lattice 2-flavor QCD at finite T [3].

Our previous investigations were mainly restricted to fermionic bulk variables like $< \overline{\psi}\psi >$, $< \overline{\psi}\gamma_5\psi >$ and the 'pion' norm

$$\langle \Pi \rangle = \Big\langle \mathrm{Tr}\Big(\mathcal{M}^{-1} \gamma_5 \mathcal{M}^{-1} \gamma_5 \Big) \Big\rangle_G / 4V.$$

$\langle\ \rangle_G$ means averaging over gauge field configurations, $V = N_\tau \cdot N_s^3$ is the number of lattice sites.

Here we present mainly results for the 'pion' mass m_π obtained from the pseudo-scalar non-singlet correlator

$$\Gamma(\tau) \equiv \frac{1}{N_s^6} \cdot \sum_{\vec{x},\vec{y}} \left\langle \mathrm{Sp}\Big(\mathcal{M}_{xy}^{-1} \gamma_5 \mathcal{M}_{yx}^{-1} \gamma_5 \Big) \right\rangle_G \ , \tag{4}$$

where Sp denotes the trace with respect to the Dirac indices.

The simulations for full QED have been carried out with the HMC method, the inversion of the matrix $\mathcal{M}$ with a BiCGstab algorithm. As in our previous work [1] for representative β–values within the confinement phase we have considered the strong coupling limit $\beta = 0$ and $\beta = 0.6,\ 0.8$.

3 Results

First let us discuss the strong coupling limit $\beta = 0$. There, for both the quenched approximation and the full theory the 'pion norm' data are compatible with a PCAC-like relation

$$\langle \Pi \rangle = \frac{C_0}{m_q} + C_1 \ , \quad m_q \to 0 \ , \tag{5}$$

where the constant $C_0 > 0$ – up to a factor – can be identified with the subtracted chiral condensate (see [4]). C_0 and C_1 turned out to differ slightly for the quenched and dynamical cases. The fits for $\kappa_c(0)$ provided for the quenched case $\kappa_c(0) = 0.2502(1)$ (nicely agreeing with the analytically known value $1/4$) and for the dynamical case a slightly shifted value 0.2450(6).

In Fig. 2 the dependence of m_π^2 on κ for the full and quenched theories is shown for an $8^3 \times 16$ lattice at $\beta = 0$. The quenched data for κ very close to κ_c were obtained by an improved estimator of m_π [4] in order to increase the signal-to-noise ratio. In both, quenched and dynamical cases we observe a dependence of m_π^2 on κ compatible with $m_\pi^2 \sim \left(1 - \frac{\kappa}{\kappa_c}\right)$, $\kappa \le \kappa_c$, which in the limit $\kappa \to \kappa_c$ transforms into the PCAC–like relation $m_\pi^2 = B \cdot m_q$, $m_q \to 0+$. The linear extrapolations of m_π^2 down to zero provide estimates for $\kappa_c(0)$ compatible with the above mentioned values. Thus, we conclude that at very strong coupling a

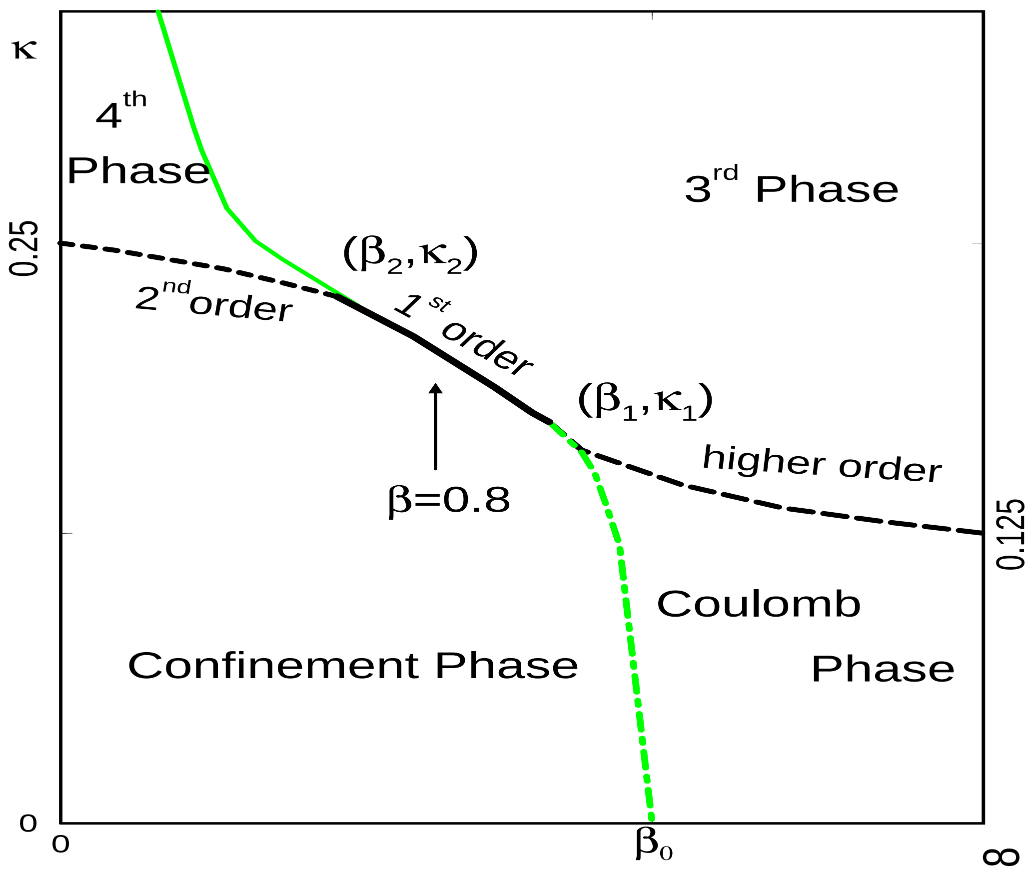

Figure 1: The phase diagram of compact lattice QED with dynamical Wilson fermions

chiral limit with a vanishing pseudo-scalar mass can be defined both in the quenched and the dynamical fermion case. The chiral transition at $\beta = 0$ is of second order.

The situation changes drastically, if we proceed to larger β–values. At $\beta = 0.8$ the quenched and dynamical cases strongly differ from each other. In the quenched case the behaviour of the 'pion norm' and of the 'pion' mass very much resembles to the results obtained in the strong coupling limit $\beta = 0$. However, in the presence of dynamical fermions the fermionic bulk variables as well as gauge observables undergo a discontinuous jump at

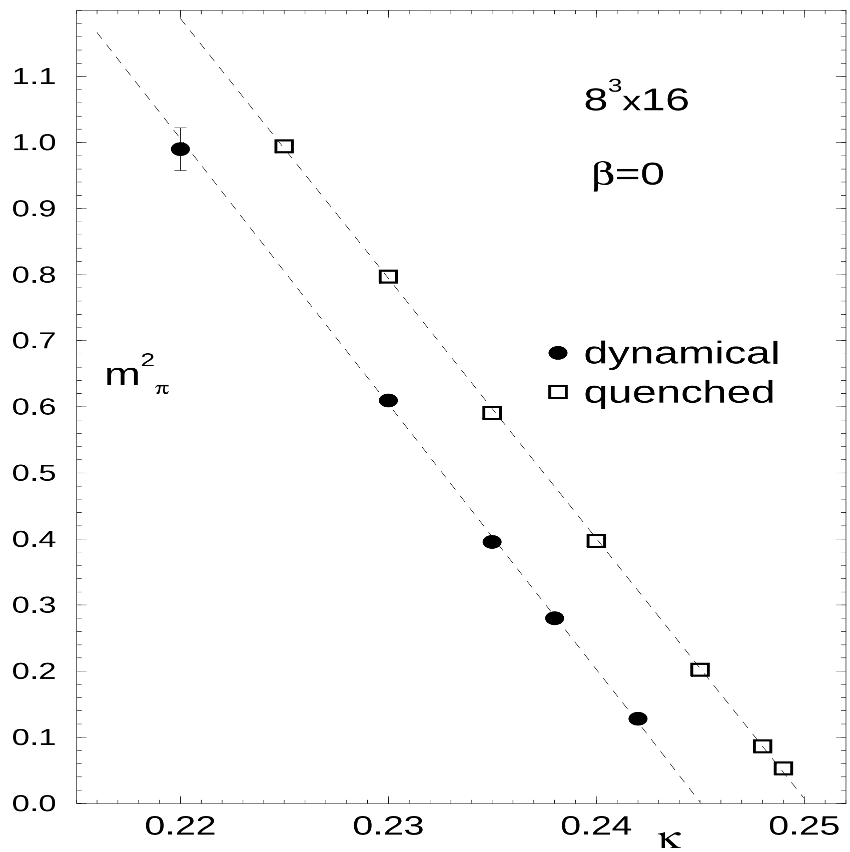

Figure 2: m_π^2 as function of κ for the quenched and full compact QED at $\beta = 0$.

$\kappa_c(0.8) = 0.1832(3)$. The discontinuity of the 'pion norm' increases with the lattice size as has been checked for sizes $8^3 \times 16$ and $16^3 \times 32$. On top of κ_c a clear metastability behaviour was observed. The 'pion' mass passes a non-vanishing minimum value at κ_c . It only slightly decreases for increasing lattice sizes. The corresponding data are plotted in Fig. 3.

We conclude that at $\beta = 0.8$ the transition becomes certainly a first order transition under the influence of the fermionic determinant. As far as the pseudo-scalar mass does not vanish the standard definition of the chiral limit does not apply.

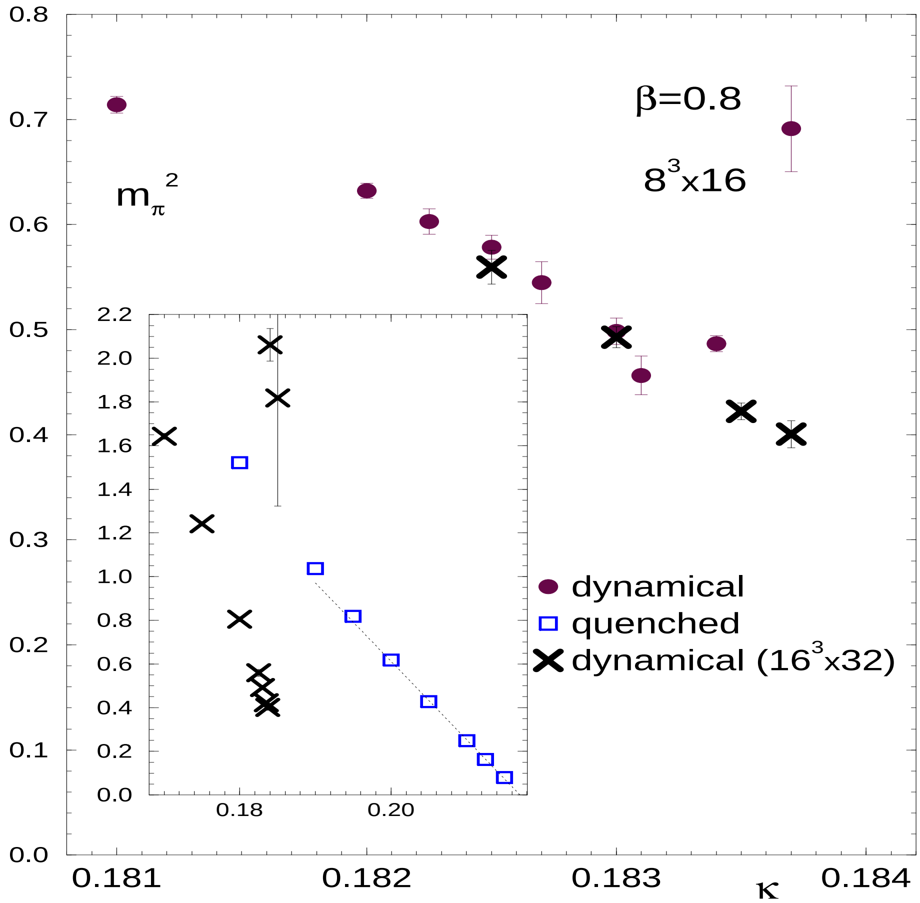

Figure 3: Same as Fig.2 but at $\beta = 0.8$. Note the scales on the inset and outer plot.

We carried out simulations also at $\beta = 0.6$. There the transition becomes weaker, the minimal 'pion' mass decreases. This is not surprising, because we are approaching a second order transition at stronger bare coupling.

4 Summary

We have studied the approach to $\kappa_c(\beta)$ for several β–values within the confinement phase of the compact lattice QED with Wilson fermions com-

paring the full theory with its quenched approximation. We have shown the importance of vacuum polarization effects due to dynamical fermions in the context of the chiral limit. In the strong coupling limit $\beta = 0$ the only effect of dynamical fermions seems to be a renormalization of the 'critical' value κ_c, $\kappa_c^{dyn} \neq \kappa_c^{quen}$. In the quenched as well as dynamical fermion case the pseudo-scalar particle becomes massless when $\kappa \to \kappa_c$. However, at $\beta = 0.8$ the presence of the dynamical ('sea') fermions drastically changes the transition. There the transition cannot be anymore associated with the zero-mass limit of a pseudo-scalar particle, in sharp contrast to the quenched case. Since the transition is first order, we can speculate about the existence of tri-critical points on the line $\kappa_c(\beta)$.

References

[1] A. Hoferichter, V.K. Mitrjushkin, M. Müller-Preussker, Th. Neuhaus and H. Stüben, Nucl. Phys. B434 (1995) 358.

[2] S. Aoki, Phys. Rev. D30 (1984) 2653; Phys. Rev. Lett. 57 (1986) 3136; Phys. Lett. 190B (1987) 140; UTHEP-318 (hep-lat/9509008)

[3] S. Aoki, T. Kaneda, A. Ukawa and T. Umemura, Nucl. Phys. B (Proc. Suppl.) 53 (1997) 438;
A. Ukawa, Nucl. Phys. B (Proc. Suppl.) 53 (1997) 106.

[4] A. Hoferichter, V.K. Mitrjushkin and M. Müller-Preussker, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 669; Z. f. Physik C74 (1997) 541.